Analysis Of Forced Transverse Vibration of A Double-Beam System With A Linear Pasternak Middle Layer Under A Travelling Distributed Load

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Abstract: This study explores the compelled oscillation of a double beam structure using the Euler-Bernoulli theory. The structure consists of two equivalent, uniform, parallel beam that are simply Supported and have a linear Pasternak middle layer. The beams' transverse responses, the coupled dynamic equations of motion of the system are simplified using finite Fourier sine integral transform. This integral transformation facilitates the decoupling of the equations, resulting in second-order differential equations. The differential transform method is subsequently used to simplify these equations further and derive the dynamics response equations. The obtained responses were analysed using Scilab tools under conditions of moving force for both beams. The impact of the moving load's speed (v), the shear variable (G), and stiffness parameter (k) on the traveling force were studied. Ultimately, different values of the travelling load, shear variable, and stiffness parameter were analysed through plots. The outcomes reveal that as the traveling load's speed increments, the absolute response magnitude of the deflection of the upper and lower beams increment as well. An upsurge in the shear parameter causes a decline in the dynamic amplitude of the upper beam, while the opposite is seen for the lower beam. It was noted that an increments in the stiffness parameter results to an increment in the dynamic amplitude of the upper beam, while the opposite is noted for the lower beam.

Keywords: Forced vibration, Euler-Bernoulli beam, Pasternak middle layer, Transverse response, Moving force.

Introduction

The problem concerning the investigation of vibrational modelling of a double beam system under moving distributed load is very essential in structural engineering application. Double-beam systems are frequently utilized in diverse engineering fields, such as construction, bridge building, and aerospace structures, where it is crucial to have a high load-bearing capacity and stiffness. Double beam system can be designed with a longer span than single beams, making them suitable for larger structures. Double beam systems exhibit more intricate vibrations under moving loads than single beam systems, which may be linked to the difficulty in solving dynamic motion equations. Compared to single-beam systems, the investigation of vibrations in double-beam systems is relatively complex, which has garnered the attention of only a small group of researchers, as noted by Michaltos et al. (1995), Abu-Hilal (2006), Li et al. (2015), and Jing et al. (2019). Over time, many problems related to this study on the vibrational reaction of beams due to travelling loads have been examined and explicit solutions have been found. Rajib et al. (2012), investigated dynamic reaction of beams upheld by a Pasternak substructure and exposed to a mobile load and mobile mass. The authors utilized modal analysis in conjunction with Fourier transverse techniques to analyze the dynamic equations governing the system. The findings demonstrated that the modal analysis results were similar to those obtained from exact analytical solutions.
Nasirshoabi and Mohammadi (2015) conducted a study focused on analyzing the compelled lateral vibration of a double-plate configuration with a flexible Pasternak interlayer. The research investigated how the inclusion of the Pasternak layer affected the driven vibration of the double-plate setup, particularly under a specific excitation loading condition. The researchers analysed the dynamic behaviour of the structure to randomly dispersed moving loads and discussed the resulting vibration due to harmonic exciting forces. Furthermore, they developed conditions for resonance and dynamic vibration absorption through their formulation. Li et al. (2016), analysed the dynamic characterization of a dual-beam structure linked by a visco-elastic middle layer. The researchers created a semi-analytical approach to examine mode shapes and natural vibration rate of the dual beam structure with a viscoelasticity layer and utilized integral modal analysis technique to calculate the beam responses. The authors investigated the impact of the viscoelastic layer and found that a rise in stiffness caused a reduction in the dynamic behavior due to the upper beam while the response corresponding to the lower beam increments. A study conducted by Koziol and Rafal (2018) on the vibrational response of duo-beam model with nonlinearized viscoelastic substructure to traveling load. The model is analysed using Fourier Analysis and Adomian's decomposition is adopted, together with the wavelet-based approximation of the outcome adopting Coiflet filters.
In their study, hammed et al. (2020), analysed the vibration behavior of a double beam structure that is elastically restrained at one end under a concentrated travelling mass. The focus is on the transverse behaviour of the structure. The authors employed a solution approach that involved a combination of the modified Struble's method, the series variable separable technique, and the differential transformation method (DTM). They investigated the impact of various mass values of the travelling load, as well as the

travelling load's speed, viscoelasticity and stiffness parameters on the responsive behaviour of the beam. The study presents and discusses the findings, which reveal that the responsive behaviour of the system was significantly influenced by these variables. The authors' analysis provides valuable insights into the model and analysis of such systems, particularly in mechanical engineering. Hammed et al. (2020), analyse the reaction of a double-beam model that is supported by an elastic Pasternak foundation under a dispersed travelling load. The focus is on the forced reaction of the system. The authors utilized the Fourier sine integral scheme and differential transformation to derive an exact solution. Based on the results, it was noted that an upsurge in the velocity of the travelling load led to a rise in the absolute vibration amplitude of both beams. In addition, the authors investigated the impact of other interacting parameters such as shear moduli and spring stiffness and concluded that these parameters have an important impact on the analysis.

In this study, the foundation linking the double beam is defined as a linear Pasternak type layer, where the Winkler layer is linear and the Pasternak layer is formulated using of linear second partial derivatives. To the best knowledge of the author, many researchers have worked on vibrational response of a double beam model with a linear and non-linear Pasternak type layer along the length of the beam under a travelling load. The results of this is that the effect of traveling distributed load have not been considered. This study is original and first attempt to approach the behavior of a double beam structure, linked by a linear variable Pasternak middle layer to a uniformly distributed traveling load using Fourier Integral Sine Transform and Differential Transform Method (D.T.M).

Mathematical Model

The study involves the analysis of a structural system consisting of a double Euler-Bernoulli beam system that is simply supported and supported elastically. This model consists of two uniform, undamped, and finite parallel beams with the equal length L and mass per unit length μ. The two beams are linked together by a Pasternak middle layer, which has a linear variable characteristic. The beams in the structural model exhibit linear elastic material behavior, and their cross-sections remain rigid and uniform along the entire length of the beams, which is confined to a single plane of geometry. Although the elastic axial deformations are ignored, the shear strain of the cross-section is considered.

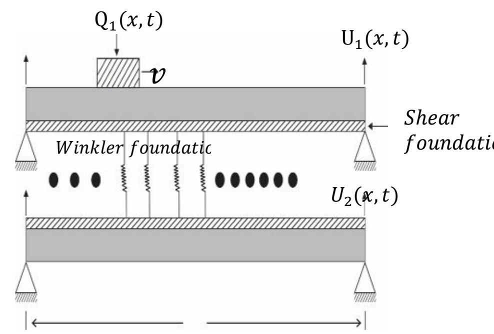

Figure 1: *A double-beam system subjected to a moving force.*

The model under a dynamic distributed load, denoted as $Q_1(x,t)$, with a mass of M and a constant velocity of v. The vibration responses of the upper beams, denoted as $U_1(x,t)$and $U_2(x,t)$respectively, are described by fourth-order Partial differential dynamic equations.

The problem under consideration is governed by the following equations:

$$EI\frac{\partial^4 U_1(x,t)}{\partial x^4} + \mu\frac{\partial^2 U_1(x,t)}{\partial t^2} + G_0\left[\frac{\partial^2 U_1(x,t)}{\partial x^2} - \frac{\partial^2 U_2(x,t)}{\partial x^2}\right] - K_0(1-\alpha x)[U_1(x,t) - U_2(x,t)] = Q_1(x,t) \quad (1)$$

$$EI\frac{\partial^4 U_2(x,t)}{\partial x^4} + \mu\frac{\partial^2 U_2(x,t)}{\partial t^2} + G_0\left[\frac{\partial^2 U_2(x,t)}{\partial x^2} - \frac{\partial^2 U_1(x,t)}{\partial x^2}\right] - K_0(1-\alpha x)[U_2(x,t) - U_1(x,t)] = 0 \quad (2)$$

where the function defining the dynamic distributed load is given by

$$Q_1(x,t) = -\frac{\rho}{\mathcal{E}}\begin{cases}\left[H\left(x-\xi+\frac{\mathcal{E}}{2}\right) - H\left(x-\xi-\frac{\mathcal{E}}{2}\right)\right], \text{for the upper beam,} \\ 0, \quad \text{for the lower beam;}\end{cases} \quad (3)$$

Where E and I are the elasticity modulus and horizontal inertia moment for the two beams, respectively, and $U_1(x,0)$ and $U_2(x,0)$ are the lateral displacement of the two beams, respectively; K_0 denotes the stiffness of the Pasternak foundation, G_0 is the shear modulus of the Pasternak foundation, ρ is the beam material's density, H is the heaviside function and δ is the dirac delta functions at point $x = vt$.

Mathematically, the boundary constraints corresponding to equations (1) and (2) can be expressed as:

$$U_1(0,t) = 0 = U_1(L,t) \quad (4)$$

$$\frac{\partial^2 U_1(0,t)}{\partial x^2} = 0 = \frac{\partial^2 U_1(L,t)}{\partial x^2} \quad (5)$$

$$U_2(0,t) = 0 = U_2(L,t) \quad (6)$$

$$\frac{\partial^2 U_2(0,t)}{\partial x^2} = 0 = \frac{\partial^2 U_2(L,t)}{\partial x^2}, \quad (7)$$

and the corresponding initial constraints are:

$$U_1(x,0) = 0 = \frac{\partial U_1(x,0)}{\partial t} \quad (8)$$

$$U_2(x,0) = 0 = \frac{\partial U_2(x,0)}{\partial t}, \quad (9)$$

Solution of the Problem

Finite Fourier Integral Sine Transform

The initial boundary-value problem outlined in the equations, a solution method needs to be employed in equations (1) - (9), the finite integral transform method was employed. This method was chosen due to its suitability in the analysis of moving load, as established by previous studies [Gbadeyan and Oni (1995), Usman et al. (2021), Mohammadi and Nasirshoabi (2015), and Hammed et al. (2020)]. The unknown transverse responses $U_1(x,t)$and $U_2(x,t)$for the upper and lower beams, respectively, were derived by applying the finite Fourier sine transform to equations (1) and (2). Considering the upper beam, equation (1) can be represented below:

$$EI\frac{\partial^4 U_1(x,t)}{\partial x^4} + \mu\frac{\partial^2 U_1(x,t)}{\partial t^2} + G_0\left[\frac{\partial^2 U_1(x,t)}{\partial x^2} - \frac{\partial^2 U_2(x,t)}{\partial x^2}\right]$$
$$-K_0(1-\alpha x)[U_1(x,t) - U_2(x,t)] = \frac{Mg}{\mathcal{E}}\left[H\left(x-\xi+\frac{\mathcal{E}}{2}\right) - H\left(x-\xi-\frac{\mathcal{E}}{2}\right)\right] \quad (10)$$

and equation (10) becomes

$$EI\frac{\partial^4 U_2(x,t)}{\partial x^4} + \mu\frac{\partial^2 U_2(x,t)}{\partial t^2} + G_0\left[\frac{\partial^2 U_2(x,t)}{\partial x^2} - \frac{\partial^2 U_1(x,t)}{\partial x^2}\right]$$
$$-K_0(1-\alpha x)[U_2(x,t) - U_1(x,t)] = 0. \quad (11)$$

The finite Fourier sine transform assumed for equations (1) and (2) is defined as:

$$\bar{U}_m(n,t) = \int_0^l U_m(x,t)\sin\frac{n\pi x}{L}dx; \qquad n = 1,2,3,\ldots; \qquad m = 1,2. \quad (12)$$

The inverse form of equation (12) is

$$U_m(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\bar{U}_m(n,t)\sin\frac{n\pi x}{L}, \quad m = 1,2. \quad (13)$$

Applying equation (12) into individual terms of equation (10), the following expression was obtained

$$EI\frac{n^4\pi^4}{L^4}\bar{U}_1(n,t) + \mu\ddot{\bar{U}}_1(n,t) + G_0\frac{n^2\pi^2}{L}[\bar{U}_2(n,t) - \bar{U}_1(n,t)]$$
$$+K_n(x)[\bar{U}_2(n,t) - \bar{U}_1(n,t)] = -2\frac{Mg}{n\pi\varepsilon}\sin^2\frac{n\pi\xi}{L}\sin^2\frac{n\pi\varepsilon}{L} \quad (14)$$

$$EI\frac{n^4\pi^4}{L^4}\bar{U}_2(n,t) + \mu\ddot{\bar{U}}_2(n,t) + G_0\frac{n^2\pi^2}{L}[\bar{U}_1(n,t) - \bar{U}_2(n,t)]$$
$$-K_n(x)[\bar{U}_1(n,t) - \bar{U}_2(n,t)] = 0. \quad (15)$$

Simplification of equations (14) and (15) gives

$$\ddot{\bar{U}}_1(n,t) + \omega_n^2\bar{U}_1(n,t) + \alpha_n\bar{U}_2(n,t) - \alpha_n\bar{U}_1(n,t) = \frac{Mg}{\mu}\sin\frac{n\pi vt}{L} \quad (16)$$

$$\ddot{\bar{U}}_2(n,t) + \omega_n^2\bar{U}_2(n,t) + \alpha_n\bar{U}_1(n,t) - \alpha_n\bar{U}_2(n,t) = 0 \quad (17)$$

where

$$\alpha_n = G_0\frac{n^2\pi^2}{\mu L^2} + \frac{K_n(x)}{\mu} \quad (18)$$

$$\omega_n^2 = \frac{EI}{\mu}\frac{n^4\pi^4}{L^4}, \quad (19)$$

where $\omega_n$is the natural angular frequency of the beam.

It should be noted that equations (16) and (17) represent a simplified version of the dynamic equations that dictate the behavior of the beams, as originally expressed in equations (1) and (2). These simplified equations are obtained by performing a finite Fourier transformation on the dynamic governing equations of the system. Essentially, the fourth order dynamic equations (1) and (2) are converted to second-order ordinary differential equations (16) and (17). To further simplify equations (16) and (17), the differential transformation method is utilized.

Differential Transform Method

The differential transform technique, which is built on the fundamental principle of transforming differential equations into algebraic equations, was employed to analyse the two differential equations (16) and (17). The differential transformation method is a well-established numerical technique that is widely recognized as a powerful and efficient tool for solving a wide range of differential equations, both uniform and non-uniform, which does not exclude partial and ordinary differential equations. The method is known for its fast convergence rates, minimal computational effort, and low calculation errors, making it a valuable tool for various engineering applications. Zhou (1986) first introduced the concept of DTM while attempting to analyse the initial boundary value problems in engineering applications. In the last few years, several researchers have employed the differential transformation method (DTM) for obtaining a variety of dynamic vibration problems. Raslan et al. (2012), investigated comprehensive research on the implementation of the differential transform method (DTM) for calculating dynamic equations that have variable coefficients. Other notable studies on vibration problems of moving load structures using DTM include those by Ho and Chen (1998), Attarnejad et al. (2017), Gbadeyan and Agboola (2012), and Gbadeyan and Hammed (2017).

Illustrating the basic idea of differential transform, the function $\overline{U}_m(n,t)$ which is analytic and having a continuous derivative in the scope of interest is considered such that

$$U_m(k) = \frac{1}{K!}\left[\frac{d^k\overline{U}_m(n,t)}{dt^k}\right]_{x=x_0}, \qquad (20)$$

where the original function is denoted by $\overline{U}_m(n,t)$,while the transformed function is denoted by $U_m(k)$. The differential inverse transform of $U_m(k)$is expressed as:

$$\overline{U}_m(n,t) = \sum_{k=0}^{\infty} U_m(k)(t-t_o)^k \quad . \qquad (21)$$

Taking equations (20) and (21) into consideration, the deduced equation is:

$$\overline{U}_m(n,t) = \sum_{k=0}^{\infty} \frac{(t-t_o)^k}{k!}\frac{d^k\overline{U}_m(n,t)}{dt^k}\bigg|_{t=0} . \qquad (22)$$

when t_0 is set to 0, equation 22 gives:

$$\overline{U}_m(n,t) = \sum_{k=0}^{\infty} \frac{t^K}{K!}\left[\frac{d^k\overline{U}_m(n,t)}{dt^k}\right]_{t=0} . \qquad (23)$$

Hence,

$$\overline{U}_m(n,t) = \sum_{k=0}^{\infty} U_m(k)t^k . \qquad (24)$$

The primary contrast between the differential transform technique and Taylor's series technique lies in the fact that the latter involves the computation of higher-order derivatives, which can be challenging, while the former employs an iterative approach that avoids the need for such computations, making it a simpler and more efficient technique for solving differential equations. In practical applications, such as the situation at hand, the function $\overline{U}_m(n,t)$ is usually a finite series, so that equation (24) can be expressed as

$$\overline{U}_m(n,t) = \sum_{k=0}^{N} U_m(k)\, t^k \qquad (25)$$

Therefore, the term $\sum_{k=N+1}^{\infty} U_m(k)\, t^k$is considered to be negligible, and the value of N is obtained in this study based on the convergence of vibration frequency.

Taking the differential transform of equation (16) and (17), the following recurrence equation was obtained:

$$(k+2)(k+1)U_1(k+2) + \omega_n^2 U_1(k) + \alpha_n U_2(k) - \alpha_n U_1(k) = \frac{Mg}{\mu}\left[\frac{1}{k!}\left(\frac{n\pi v}{k}\right)\sin\left(\frac{k\pi}{2}\right)\right] \qquad (26)$$

$$(k+2)(k+1)U_2(k+2) + \omega_n^2 U_2(k) + \alpha_n U_1(k) - \alpha_n U_2(k) = 0 . \qquad (27)$$

The corresponding recurrence relation of equation (26) and (27) are respectively expressed as,

$$U_1(k+2) = \frac{1}{(k+2)(k+1)}\left[\frac{Mg}{\mu}\left[\frac{1}{k!}\left(\frac{n\pi v}{k}\right)\sin\left(\frac{k\pi}{2}\right)\right] - \omega_n^2 U_1(k) - \alpha_n U_2(k) + \alpha_n U_1(k)\right] \qquad (28)$$

$$U_2(k+2) = \frac{1}{(k+2)(k+1)}[-\omega_n^2 U_2(k) - \alpha_n U_1(k) + \alpha_n U_2(k)]\,. \qquad (29)$$

The following initial conditions based on DTM theorems at $t = 0$ are defined:

$$U_m = (0); \qquad \frac{dU_m}{dt} = 0; m = 1,2\,. \qquad (30)$$

Thus, the corresponding transformed boundary conditions are,

$$U_1(0) = 0 = U_1(1) \qquad (31)$$
$$U_2(0) = 0 = U_2(1)\,. \qquad (32)$$

Hence, for $k = 0,\ 1, 2, 3, \ldots$ in equations (28) and (29) the following results are obtained:

For; $k = 0$

$$U_1(2) = 0 \qquad (33)$$
$$U_2(2) = 0\ . \qquad (34)$$

$k = 1$

$$U_1(3) = \frac{1}{3!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right) \qquad (35)$$
$$U_2(3) = 0\ . \qquad (36)$$

$k = 2$

$$U_1(4) = 0 \qquad (37)$$
$$U_2(4) = 0\ . \qquad (38)$$

$k = 3$

$$U_1(5) = \frac{1}{5!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\left(\frac{n\pi v}{L}\right)^2 + \omega_n^2 + \alpha_n\right] \qquad (39)$$
$$U_2(5) = -\frac{\alpha_n}{5!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\,. \qquad (40)$$

$k = 4$

$$U_1(6) = 0 \qquad (41)$$
$$U_2(6) = 0\ . \qquad (42)$$

$k = 5$

$$U_1(7) = \frac{1}{7!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\left(\frac{n\pi v}{L}\right)^4 - \omega_n^2\left(\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right) + \alpha_n^2 + \alpha_n\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right]\right] \qquad (43)$$

$$U_2(7) = \frac{\alpha_n}{7!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\omega_n^2 - 1\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right] - \alpha_n\right]\ . \qquad (44)$$

The undefined functions $\bar{U}_m(n,t), m = 1, 2$ are then represented as the inverse differential transform given by:

$$\bar{U}_m(n,t) = \sum_{k=0}^{N} U_m(k)t^K\ ; \qquad m = 1,2\ . \qquad (45)$$

substituting equations (33) – (44) respectively into equation (45) yields

$$\bar{U}_1(n,t) = \frac{1}{3!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)t^3 + \frac{1}{5!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\left(\frac{n\pi v}{L}\right)^2 + \omega_n^2 + \alpha_n\right]t^5 + \frac{1}{7!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\left(\frac{n\pi v}{L}\right)^4 - \omega_n^2\left(\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right) + \alpha_n^2 + \alpha_n\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right]\right]t^7 + \cdots \qquad (46)$$

$$\bar{U}_2(n,t) = -\frac{\alpha_n}{5!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)t^5 + \frac{\alpha_n}{7!}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\omega_n^2 - 1\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right] - \alpha_n\right]t^7 + \cdots\ . \qquad (47)$$

Further simplification of equations (46) and (47) yields;

$$\bar{U}_1(n,t) = \frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\frac{t^3}{3!} + \frac{1}{5!}\left[\left(\frac{n\pi v}{L}\right)^2 + \omega_n^2 + \alpha_n\right]t^5 + \frac{1}{7!}\left[\left(\frac{n\pi v}{L}\right)^4 - \omega_n^2\left(\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right) + \alpha_n^2 + \alpha_n\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right]\right]t^7 + \cdots\right] \quad (48)$$

$$\bar{U}_2(n,t) = \frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[-\frac{\alpha_n t^5}{5!} + \frac{\alpha_n}{7!}\left[\omega_n^2 - 1\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right] - \alpha_n\right]t^7 + \cdots\right] \quad (49)$$

respectively.
plugging equations (48) and (49) into equation (13) for $m = 1$ and $m = 2$ yields the following equations:

$$U_1(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[\frac{t^3}{3!} + \frac{1}{5!}\left[\left(\frac{n\pi v}{L}\right)^2 + \omega_n^2 + \alpha_n\right]t^5 + \frac{1}{7!}\left[\left(\frac{n\pi v}{L}\right)^4 - \omega_n^2\left(\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right) + \alpha_n^2 + \alpha_n\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right]\right]t^7 + \cdots\right]\sin\frac{n\pi x}{L} \quad (50)$$

$$U_2(x,t) = \frac{2}{L}\sum_{n=1}^{\infty}\frac{Mg}{\mu}\left(\frac{n\pi v}{L}\right)\left[-\frac{\alpha_n t^5}{5!} + \frac{\alpha_n}{7!}\left[\omega_n^2 - 1\left[\left(\frac{n\pi v}{L}\right)^2 - \omega_n^2 + \alpha_n\right] - \alpha_n\right]t^7 + \cdots\right]\sin\frac{n\pi x}{L} \quad (51)$$

respectively.
Therefore, equations (50) and (51) describe the dynamic behavior of Euler-Bernoulli beams that have a dynamic linear variable Pasternak middle layer. The beams in consideration are supported Simply at both ends and exposed to a traveling uniform partially distributed load

Results and Discussion

Numerical Results

The Pasternak middle layer that supports the two beams can have varying properties, as previously mentioned. A uniform partial distributed moving force was applied to the beams, and to investigate the impact of the layer's shear stiffness and other beam parameters, equations (50) and (51) were simulated using SCILAB based on the analytical findings. These numerical computations were conducted for both beams, and values from Abul-Hilal (2006) and adopted by Hammed et al. (2020) were utilized for comparison purposes.

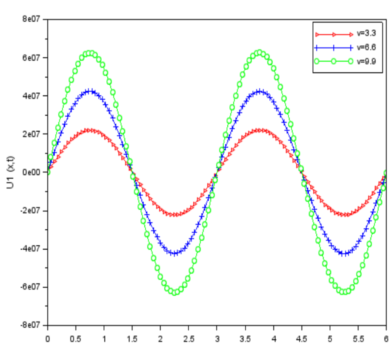

Figure 2(a): A graph depicting the variation of speed and its impact on the upper beam's deflection.

$\mu = 0.075$; $EI = 16{,}000$; $g = 10$; $L = 6$; $\varepsilon_0 = 0.10, 0.20, 0.30, 0.35$; $x = 3$; $k = 10$; $\pi = \frac{22}{7}$; $t = 0.5$.

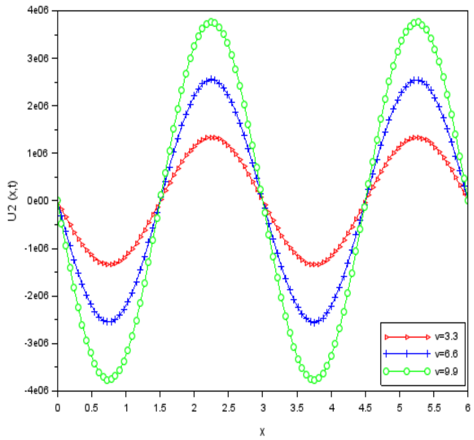

Figure 2(b): A graph illustrating the variation of speed and its impact on the lower beam's deflection.

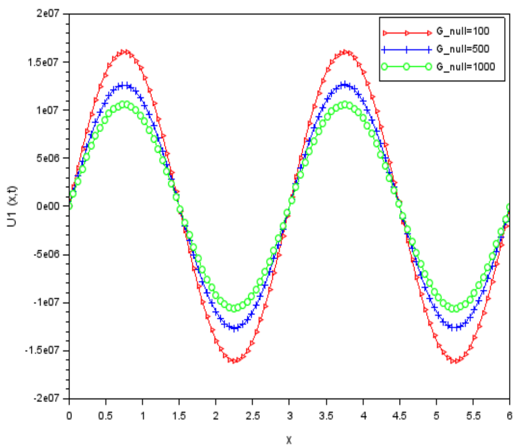

Figure 3(a): A graph depicting the variation of the shear modulus and its impact on the upper beam's deflection.

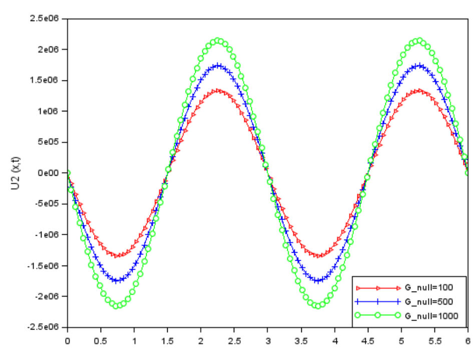

Figure 3(b): A graph depicting the variation of the shear modulus and its impact on the lower beam beam's deflection.

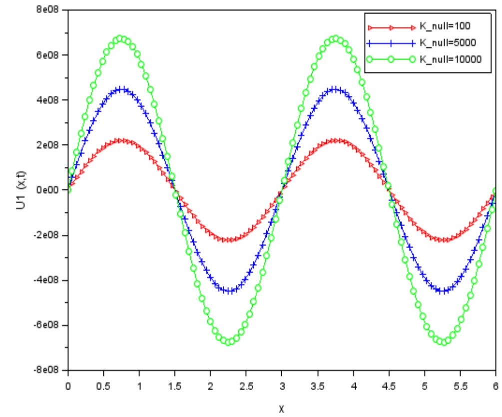

Figure 4(a): A graph depicting the variation of the stiffness parameter and its impact on the upper beam's deflection.

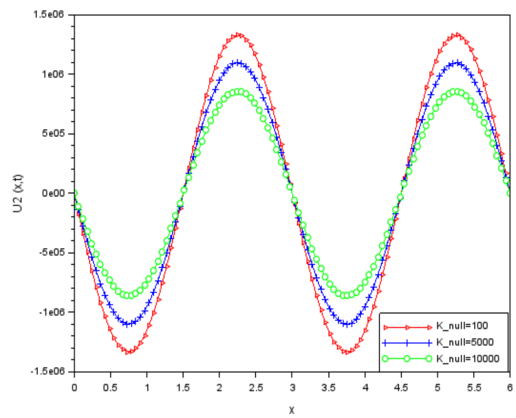

Figure 4(b): A graph depicting the variation of the stiffness parameter and its impact on the lower beam's deflection.

Discussion of Results

The investigation analysed the impact of several parameters, such as the velocity of the mobile load, shear modulus element, and stiffness parameter, on the responsiveness. The individual and coupling impact of these parameters were observed and discussed extensively.

The influence of the traveling load speed on the upper beam is depicted in Figure 2(a), where an augmentation in the speed shows an increment in the dynamic magnitude of the upper beam. On the other hand, an increment in the velocity of the traveling load causes an increment in the dynamic amplitude of the lower beam, as shown in Figure 2(b). Nevertheless, the upper beam exhibits a higher deflection than the lower beam.

The vibrational response of the upper beam with varying elastic modulus values is depicted in Figure 3(a). The graph indicates that an escalation in the elastic modulus layer value causes a reduction in the dynamic amplitude of the upper beam. Furthermore, Figure 3(b) exhibits the impact of different shear modulus layer values on the lower beam, wherein an upsurge in the shear modulus leads to an increment in the dynamic magnitude of the lower beam

Figure 4(a) displays the absolute response amplitudes resulting from variations in the stiffness parameter. It should be emphasised that augmenting the stiffness parameter values results in an elevation of the dynamic amplitude of the upper beam deflection. Figure 4(b) illustrates the impact of different stiffness parameter values on the lower beam. It can be noted that enhancing the stiffness parameter value causes a reduction in the dynamic amplitude of the lower beam.

Conclusion

The study utilized numerical computations to investigate how the two beams responded dynamically to the applied moving force at different velocities. The results indicated that as the velocity of the moving force increased, there was a corresponding increase in the deflections of the beams. Additionally, the computations were conducted for various stiffness parameters, and the outcomes indicated that the upper beam's transverse deformation increased while less deflection occurred in the lower beam with an increment in the stiffness parameter. Moreover, the numerical findings revealed that for varying values of the elastic modulus, an increase in the elastic modulus led to a decrement in the absolute vibration amplitude of the upper beam, while an increment in absolute vibration amplitude of the lower beam was observed.

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